Degeneracy and strategies of long baseline and reactor experiments *

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Assuming that the JPARC experiment measures the oscillation probabilities $P(\nu_\mu \to \nu_e)$ and $P(\bar{\nu}_\mu \to \bar{\nu}_e)$ at $|\Delta m^2_{31}|L/4E = \pi/2$, I discuss what kind of extra experiment (long baseline or reactor) can contribute to determination of θ_{13} and the CP phase δ.

1. Introduction

Determination of the unknown oscillation parameters θ_{13} and δ is the important object of future neutrino experiments. It has been known that even if the values of the oscillation probabilities $P(\nu_\mu \to \nu_e)$ and $P(\bar{\nu}_\mu \to \bar{\nu}_e)$ are exactly given we cannot determine uniquely the values of the oscillation parameters due to three kinds of parameter degeneracies: the intrinsic (θ_{13}, δ) degeneracy, the degeneracy of $\Delta m^2_{31} \leftrightarrow -\Delta m^2_{31}$, and the degeneracy of $\theta_{23} \leftrightarrow \pi/2 - \theta_{23}$. Each degeneracy gives a twofold ambiguity, so in general we have an eightfold ambiguity. These degeneracies cause a problem in determination of θ_{13} and δ, and we have to take into account these ambiguities in handling the data in future long baseline experiments. Here I will discuss only the oscillation probabilities without details analysis of statistical and systematic errors.

2. Determination of $\sin^2 2\theta_{13}$

There is a way to overcome the ambiguity due to the intrinsic degeneracy. Namely, if one performs a long baseline experiment at the oscillation maximum (i.e., with $|\Delta m^2_{31}L/4E| = \pi/2$), then it is reduced to the $\delta \leftrightarrow \pi - \delta$ ambiguity.

Thus, if the JPARC experiment measures the oscillation probabilities $P(\nu_\mu \to \nu_e)$, $P(\bar{\nu}_\mu \to \bar{\nu}_e)$ and $P(\nu_\mu \to \nu_\mu)$ at the oscillation maximum with an approximately monoenergetic beam, then the possible solutions in the $(\sin^2 2\theta_{13},\ 1/s^2_{23})$ plane [1] given by JPARC look like either Fig. 1(a) or Fig. 1(b), depending on whether $\cos^2 2\theta_{23} \ll \mathcal{O}(0.1)$ or $\cos^2 2\theta_{23} \simeq \mathcal{O}(0.1)$, where the difference between the case with normal hierarchy ($\Delta m^2_{31} > 0$) and the one with inverted hierarchy ($\Delta m^2_{31} < 0$) is small because the matter effect is small in the JPARC experiment.

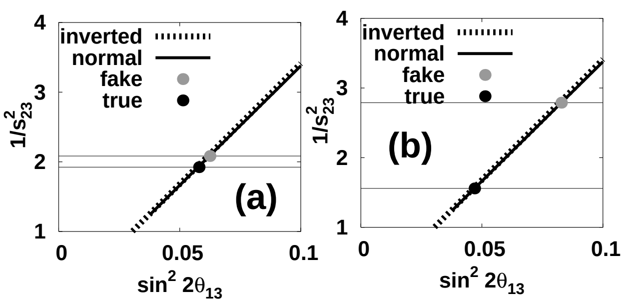

Figure 1. The θ_{23} ambiguity which could arise after the JPARC measurements of $P(\nu_\mu \to \nu_e)$, $P(\bar{\nu}_\mu \to \bar{\nu}_e)$ and $P(\nu_\mu \to \nu_\mu)$ at the oscillation maximum. (a) The case of $\cos^2 2\theta_{23} \ll \mathcal{O}(0.1)$. (b) The case of $\cos^2 2\theta_{23} \simeq \mathcal{O}(0.1)$.

If $\cos^2 2\theta_{23} \ll \mathcal{O}(0.1)$ then the values of θ_{13} and θ_{23} are close to each other for all the solutions, and the ambiguity is not serious as far as θ_{13} and θ_{23} are concerned (cf. Fig.1(a)). On the other hand, if $\cos^2 2\theta_{23} \simeq \mathcal{O}(0.1)$ then the θ_{23} ambiguity has to be resolved to determine θ_{13} and θ_{23} to good precision (cf. Fig.1(b)). In this case

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there are three potential possibilities to resolve this θ_{23} ambiguity: (a) reactor experiments, (b) the $\nu_\mu \to \nu_e$ (or $\nu_e \to \nu_\mu$) channel of accelerator long baseline experiments and (c) the so-called silver channel $\nu_e \to \nu_\tau$.

A reactor experiment measures $P(\bar{\nu}_e \to \bar{\nu}_e)$ which depends only on θ_{13} to a good approximation, so that it gives a constraint as a vertical band in the $(\sin^2 2\theta_{13}, 1/s^2_{23})$ plane (cf. Fig.2(a)). Thus, if the experimental error in the reactor experiment is smaller than the difference between the true and fake values of θ_{13}, then the reactor experiment can solve the θ_{23} ambiguity. To resolve the θ_{23} ambiguity at a high confidence level, it is necessary for a reactor experiment to have high sensitivity (e.g., $\left(\sin^2 2\theta_{13}\right)_{\text{sensitivity}} \lesssim 0.01$). It is known [2] that the sensitivity in reactor experiments is bounded from below by the uncorrelated systematic error $\sigma_{\rm u}$ of the detector:

$$\left(\sin^2 2\theta_{13}\right)_{\text{sensitivity}} \geq \text{const.}\sigma_{\rm u}, \tag{1}$$

where const. depends on the numbers of the reactors and the detectors, and is equal to 2.8 at 90%CL in the case with one reactor and two detectors. Eq. (1) implies that it is necessary to reduce $\sigma_{\rm u}$ in order to improve the sensitivity. If the uncorrelated systematic error $\sigma_{\rm u}$ of the detectors is independent of the number of the detectors, then there is a way to reduce $\sigma_{\rm u}$. In the case with one reactor, if one puts N identical detectors at the near site and N identical detectors at the far site, where all these detectors are assumed to have the same uncorrelated systematic error $\sigma_{\rm u}$, then the lower bound of the sensitivity becomes

$$\text{lower bound of } \left(\sin^2 2\theta_{13}\right)_{\text{sensitivity}} = \frac{2.8}{\sqrt{N}}\,\sigma_{\rm u}.$$

Hence it follows theoretically that the more identical detectors one puts, the better sensitivity one gets. The assumption that $\sigma_{\rm u}$ is independent of N may not be satisfied in reality, but if the dependence of $\sigma_{\rm u}$ on N is weaker than $\sqrt{N}$, $\sigma_{\rm u}/\sqrt{N}$ decreases as N increases. This possibility should be seriously thought about to improve the sensitivity in the future reactor experiments.

The $\nu_\mu \to \nu_e$ (or $\nu_e \to \nu_\mu$) channel gives poor resolution in general, as far as the θ_{23} ambiguity is concerned, because the curves given by this channel intersect almost in parallel with the JPARC line and the experimental significance to reject the wrong hypothesis is expected to be small (cf. Fig.2(b),(b')). The situation changes very little even if one uses $\bar{\nu}_\mu \to \bar{\nu}_e$ (or $\bar{\nu}_e \to \bar{\nu}_\mu$).

On the other hand, the silver channel $\nu_e \to \nu_\tau$ offers a promising possibility to resolve the θ_{23} ambiguity, because the curves given by the silver channel intersect with the JPARC line almost perpendicularly and the experimental significance to reject the wrong hypothesis is expected to be large (cf. Fig.2(c)).

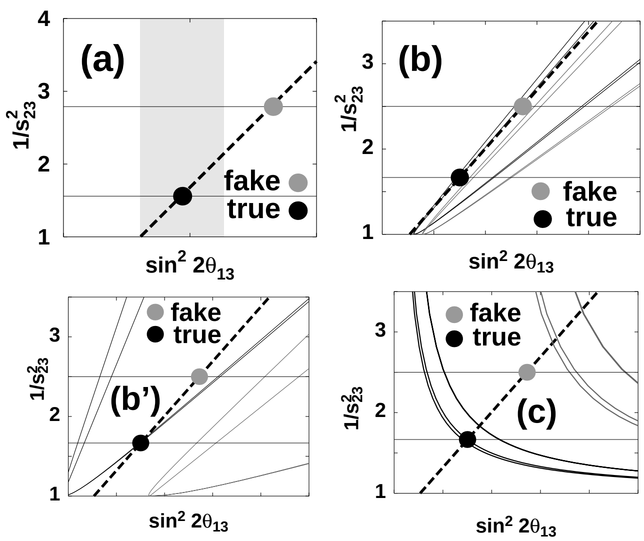

Figure 2. (a) The case of a reactor experiment, which gives the constraint only on $\sin^2 2\theta_{13}$ (the shadowed region). (b,b') The case of the $\nu_\mu \to \nu_e$ (or $\nu_e \to \nu_\mu$) channel with the baseline L=730km and the neutrino energy E=6GeV ((b)), E=1GeV ((b')). (c) The case of the $\nu_e \to \nu_\tau$ channel with L=3000km and E=12GeV. The dashed line is the constraint by the JPARC measurements of $P(\nu_\mu \to \nu_e)$ and $P(\bar{\nu}_\mu \to \bar{\nu}_e)$, where the two hierarchical patterns are identified for simplicity because the difference between them is very small, and the thin black and gray curves stand for those by the third experiment with correct and wrong assumptions on the mass hierarchy. See [1] for details of the figures.

3. Determination of δ

If the JPARC experiments for $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ are performed at the oscillation maximum, then the value of $\sin\delta$ suffers from the two ambiguities, i.e., those due to $\mathrm{sgn}(\Delta m^2_{31})$ and $\theta_{23} \leftrightarrow \pi/2 - \theta_{23}$, since the $\delta \leftrightarrow \pi - \delta$ ambiguity does not affect the value of $\sin\delta$. To simplify the discussions, let me discuss the case where the reference value of δ is zero. In this case the fake values $\sin\delta'|_{\mathrm{sgn}(\Delta m^2_{31})}$ due to the $\mathrm{sgn}(\Delta m^2_{31})$ ambiguity and $\sin\delta'|_{\mathrm{sgn}(\cos 2\theta_{23})}$ due to the $\theta_{23} \leftrightarrow \pi/2 - \theta_{23}$ ambiguity satisfy the following for the JPARC case [3]:

$$\begin{aligned} \sin\delta'|_{\mathrm{sgn}(\Delta m^2_{31})} &\simeq -2.2\sin\theta_{13}, \\ \left|\sin\delta'|_{\mathrm{sgn}(\cos 2\theta_{23})}\right| &\lesssim \frac{1}{500}\frac{1}{\sqrt{\sin^2 2\theta_{13}}}. \end{aligned}$$

This suggests that the θ_{23} ambiguity does not cause a serious problem for $\sin^2 2\theta_{13} \gtrsim 10^{-3}$, while the one due to the $\mathrm{sgn}(\Delta m^2_{31})$ ambiguity does. In Fig.3 the sensitivity to CP violation is given at 3σ by a semi-quantitative analysis for the JPARC experiment in the case of (a) $\Delta m^2_{31} > 0$ and (b) $\Delta m^2_{31} < 0$, taking into account of the ambiguity due to $\mathrm{sgn}(\Delta m^2_{31})$. The the dashed lines in the figures are given by $\sin\delta = 0$ for the correct assumption, and by $\sin\delta = \pm 2.2\sin 2\theta_{13}$ for the wrong assumption. In the region bounded by the black (with the correct assumption on the mass hierarchy) or grey (with the wrong assumption) curves, JPARC cannot claim CP violation to be nonzero. From this, therefore, we see that it is important to resolve the $\mathrm{sgn}(\Delta m^2_{31})$ ambiguity to determine the precise value of δ.

To resolve the $\mathrm{sgn}(\Delta m^2_{31})$ ambiguity, it is easy to see that long baseline experiments with long baselines ($\gtrsim$1000km) are advantageous, since the matter effect $\sqrt{2}G_F N_e$ for the density $\rho \simeq 3g/cm^3$ is something like 1/2000km. What is not trivial to see is the fact that a long baseline experiment with *lower* neutrino energy is advantageous for the *same* baseline L, as long as the energy E satisfies $\left|\Delta m^2_{31}L/4E\right| < \pi$ [1]. Hence, although the NOvA experiment has a baseline shorter than 1000km, if it is performed with lower energy (e.g., 1GeV), then it may have a chance to resolve the $\mathrm{sgn}(\Delta m^2_{31})$ ambiguity.

4. Summary

If $\cos^2 2\theta_{23} \simeq \mathcal{O}(0.1)$ then it is important to resolve the θ_{23} ambiguity to determine the value of θ_{13} by an experiment other than JPARC. On the other hand, to determine the value of δ, it is important to resolve the $\mathrm{sgn}(\Delta m^2_{31})$ ambiguity irrespective of the value of θ_{13}.

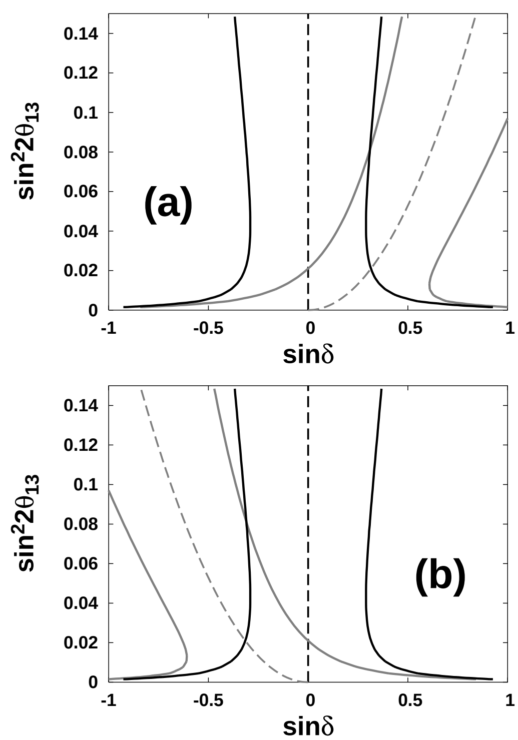

Figure 3. The sensitivity to CP violation at 3σ in the case of the JPARC experiment. The black (grey) curves give the sensitivity to CP violation with the correct (wrong) assumption on the mass hierarchy when $\Delta m^2_{31} > 0$ (a) or when $\Delta m^2_{31} < 0$ (b).

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